Near Optimal Power and Rate Control of Multi-hop Sensor Networks with Energy Replenishment: Basic Limitations with Finite Energy and Data Storage

Zhoujia Mao, Can Emre Koksal, Ness B. Shroff
E-mail: maoz@ece.osu.edu, koksal@ece.osu.edu, shroff@ece.osu.edu

***Abstract*—Renewable energy sources can be attached to sensor nodes to provide energy replenishment for extending the battery life and prolonging the overall lifetime of sensor networks. For networks with replenishment, conservative energy expenditure may lead to missed recharging opportunities due to battery capacity limitations, while aggressive usage of energy may result in reduced coverage or connectivity for certain time periods. Thus, new power allocation schemes need to be designed to balance these seemingly contradictory goals, in order to maximize sensor network performance. In this paper, we study the problem of how to jointly control the data queue and battery buffer to maximize the long-term average sensing rate of a wireless sensor network with replenishment under certain QoS constraints for the data and battery queues. We propose a unified algorithm structure and analyze the performance of the algorithm for all combinations of finite and infinite data and battery buffer sizes. We also provide a distributed version of the algorithm with provably efficient performance.**

I. Introduction

Wireless sensor networks have been widely used in monitoring [1], maintenance [2], and environmental sensing [3]. The lack of easy access to a continuous power source and the limited lifetime of batteries have hindered the wide-scale deployment of such networks. However, new and exciting developments in the area of renewable energy [4] [5] provide an alternative to a limited power source, and may help alleviate some of the deployment challenges. These renewable sources of energy could be attached to the nodes and would typically provide energy replenishment at a slow rate (compared to the rate at which energy is consumed by a continuous stream of packet transmissions) that could be variable and dependent on the surroundings.

Energy management in networks equipped with renewable sources is substantially and qualitatively different from energy management in traditional networks. For example, conservative energy expenditure could lead to (i) very long delays because the energy is not being fully used to transmit at high enough data rates, and (ii) missed recharging opportunities because the battery buffer is full. On the other hand, an over aggressive use of energy may lead to lack of coverage or connectivity for certain time periods. Further, if the battery of a node discharges completely, it could be temporarily incapable of transferring time-sensitive data. This may have undesirable consequences for many applications. Thus, new techniques and protocols must be developed for networks with replenishment to balance these seemingly contradictory goals.

In this paper, we first consider a single link communication, in which the transmitting node has a data buffer that holds the incoming variable-rate sensing data and a battery buffer, which is being replenished at a variable rate. In our model, we allow any combination of finite and infinite data and battery buffer sizes. If the data buffer size is infinite, the concern is the stability of the data queue, while for finite data buffer, excessive data losses is the undesirable event. Likewise for the battery buffer, frequent occurrences of battery discharge should be avoided. We investigate the problem of maximizing the long-term average sensing rate, subject to the constraints on the stability of the data queue (or the desired data loss ratio when the data buffer size is finite) and the desired rate of visits to zero battery state. We provide a simple and unified joint rate control and power management framework, and show that the performance of our scheme is close to optimum. We then extend our algorithm to the multihop network with multiple source destination flows under the primary or node-interference model that has been shown to be a good model for Bluetooth of FH-CDMA systems.

While the problem of energy management in sensor networks has seen considerable attention, there have been few works [6], [7], [8], [9], [10], [11], [12], [13] that also include energy replenishment. In [6], the authors consider the problem of dynamic node activation in rechargeable sensor networks. They provide a distributed threshold policy that achieves a performance within a certain factor of the optimal solution for a set of sensors whose coverage area overlap completely. In [7], the authors study the problem of computing the lexicographically maximum data collection rate for each node such that no node will be out of energy. In [8], the authors consider the problem of energy-aware routing with distributed energy replenishment. They provide an algorithm that achieves a logarithmic competitive ratio and is asymptotically optimal with respect to the number of nodes in the network. Prior work on power allocation for wireless networks without replenishment has been widely studied, e.g., [14], [15], [16]. In [15], [16] the authors assume that the data buffer is large enough so that packet loss does not occur and provide a dynamic programming based solution. In [14], the authors develop approximate algorithms to minimize the average allocated

power, or maximize the throughout given average power constraint, and at the same time keep the data queue stable. The approximation improves at the cost of increasing the data queue length and queueing delay. Most of these works assume a constant energy supply, i.e., there is no battery issue. The idea of [12] and this paper are motivated by [14]. We both model energy replenishment and consider jointly managing the data and battery buffers. This coupling between the data and battery buffers is what makes the problem notoriously difficult to solve using standard optimization based approaches. For example, unlike [14] that utilizes the fact that a static allocation policy is optimal, it is not even clear whether a static policy would even be optimal in our setting. Specifically, with a battery buffer, there is an additional energy constraint that the allocated energy should be within the battery state, and this constraint is even more difficult than the average power constraint. In [12], the authors consider infinite data buffer and finite battery buffer sizes. They assume that the replenishing process is i.i.d (for non i.i.d process, they claim that a K slot analysis can be applied, which is complex and depends on the network size), and show that the probability of battery state being less than the peak power or close to the full battery state vanishes as the battery size grows, under an index policy. In this paper, we construct a framework that accommodates all combinations of finite and infinite data and battery buffer sizes by defining minimum number of virtual queues in a general format. In addition to the constraint on the stability of the data queue (constraint on the data loss ratio when the data buffer size is finite), we also have a constraint on the frequency of battery discharge. Rather than assuming an i.i.d replenishing process as in [12], we allow for a general replenishment process without assuming ergodicity and carefully design the virtual queues and explore the relations between actual queues and virtual queues to show that our algorithm is efficient. Further, we design a distributed algorithm in detail that is based on the notion of imperfect scheduling first developed in [17].

The main intellectual contributions and challenges of the paper are as follows:

- We formulate a sensing rate maximization problem with QoS constraints on both the data and battery queues. Due to the coupling between the battery and data buffers, a stationary policy could be suboptimal, hence traditional resource allocation techniques do not directly apply. Further, dynamic programming based solutions result in prohibitively high complexity, even for the single-link scenario. Nonetheless, we are able to develop a simple and unified framework for all combinations of finite and infinite data and battery buffer sizes, and our algorithm is provably efficient.
- We extend the algorithm from a single-link case to a multi-hop network under node-exclusive interference model with practical settings and develop an efficient distributed algorithm based on imperfect scheduling.

II. Single Link Model

We first consider a single link control model in this paper, as illustrated in Figure 1. We assume a time slotted system and in time slot t, the amount of data available for the sensor node to sense is denoted by $A(t)$, which is upper bounded by $A_{\max}$, ($0 < A_{\max} < \infty$). In the same time slot, the actual amount of data the node senses and places in the data buffer is $R(t)$. The amount of energy expended by the node at time t for data transmission is $P(t)$, which is upper bounded by P_{peak}, ($0 < P_{\text{peak}} < \infty$) and the achievable data rate at that power level is $\mu(P(t))$, where we assume $\mu(\cdot)$ to be monotonically increasing, reversible and differentiable on the half real line $\Re^+ \bigcup \{0\}$. The node has a battery of size B_b (either $B_b < \infty$ or $B_b = \infty$) with zero initial battery state. We let $r(t)$ denote the amount of replenishment energy arriving to the node at time t. The energy state of the battery and the data state of the data buffer at time t is given by $q_b(t)$ and $q_d(t)$, respectively. The data buffer has size B_d (again, either $B_d < \infty$ or $B_d = \infty$). Under this setting, we describe the objective in the following section.

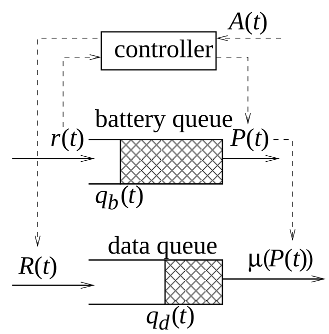


Fig. 1. Single Link Control Model

A. Problem Formulation

Our general objective is to maximize the long-term average sensing rate, subject to the QoS constraints on both data and battery queues:

$$(A) \qquad \max_{P,R} \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t)$$

$$\text{s.t. } q_d(t+1) = \min\left[\Big(q_d(t) - \mu\big(P(t)\big)\Big)^+ + R(t), B_d\right], \tag{1}$$

$$q_b(t+1) = \min\Big[q_b(t) - P(t) + r(t), B_b\Big], \tag{2}$$

$$0 \le R(t) \le A(t), \tag{3}$$

$$0 \le P(t) \le \min\Big[q_b(t), P_{\text{peak}}\Big], \tag{4}$$

$$p_o \le \eta_o, \tag{5}$$

$$\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_d(t) < \infty \ (B_d = \infty), \text{ or}$$

$$p_d \le \eta_d \ (B_d < \infty), \tag{6}$$

where $(\cdot)^+ = \max[\cdot, 0]$, $R = \{R(0), R(1), \ldots, R(T-1), \ldots\}$ is the actual sensing data vector, $P = \{P(0), P(1), \ldots, P(T-1), \ldots\}$ is the allocated power vector, and

$$p_d = \limsup_{T\to\infty} \frac{\sum_{t=0}^{T-1} D(t)}{\sum_{t=0}^{T-1} R(t)}, \tag{7}$$

$$p_o = \limsup_{T\to\infty} \frac{\sum_{t=0}^{T-1} I_o(t)}{T} \tag{8}$$

are the long-term data loss ratio with an upper bound η_d, and the frequency of visits to zero battery state with given threshold η_o respectively, where

$$D(t) = \left(\left(q_d(t) - \mu\big(P(t)\big)\right)^+ + R(t) - B_d\right)^+, \tag{9}$$

$$\begin{aligned} I_o(t) &= \mathbf{1}_{\textit{battery state '0' is visited from higher states in slot } t} \\ &= \begin{cases} 0 & \text{if } P(t) = 0 \text{ or } P(t) < q_b(t) \\ 1 & \text{otherwise} \end{cases} \end{aligned} \tag{10}$$

are the amount of data loss in slot and the indicator that the battery discharges completely in time slot t, respectively. Note that, if the battery is completely discharged at time t and there is no replenishment at slot t, we do not consider a complete discharge event occurring at time $t+1$ as well. Further note that, we do not assume ergodicity of the system parameters, but if they are ergodic, then p_o represents the actual probability of a complete discharge event as $t \to \infty$.

In Problem (A), Constraints (1) and (2) describe how the data and battery queues evolve, respectively. Especially, if $B_d = \infty$, (1) can be simplified to

$$q_d(t+1) = \left(q_d(t) - \mu\big(P(t)\big)\right)^+ + R(t), \tag{1$'$}$$

and if $B_b = \infty$, (2) can be simplified to

$$q_b(t+1) = q_b(t) - P(t) + r(t). \tag{2$'$}$$

Constraint (3) bounds the actual amount of sensed data $R(t)$ by the amount of available data $A(t)$ in slot t. Constraint (4) states that we cannot oversubscribe the energy that is unavailable in the battery nor can we exceed the peak power level. Constraint (5) is the battery QoS constraint η_o of the desired battery discharge rate. Constraint (6) is the QoS constraint for data queue: if $B_d = \infty$, we need to keep the data queue stable, and if $B_d < \infty$, the data loss ratio is required under a given threshold η_d.

Problem (A) has an inventory control structure, and typically such a structure can be solved optimally using dynamic programming, albeit with high complexity. Furthermore, depending on the exogenous processes $\{A(t),\ t \geq 0\}$ and $\{r(t),\ t \geq 0\}$, Problem (A) may not have a feasible solution, i.e., there exists no power allocation policy $\{P(t),\ t \geq 0\}$ that satisfies all constraints simultaneously. In this paper, our purpose is to develop a simple algorithm, which performs arbitrarily close to the performance of the optimal power allocation, whenever a feasible solution exists. To achieve that purpose, we do the following:

- define *virtual queues* for both the data and battery buffer to avoid the difficulties involved in dealing with the data loss and battery discharge probability directly. We show that keeping the virtual queues stable ensures that the constraints on p_d and p_o are met.
- design a power allocation scheme based on simple index policies and show that our scheme keeps both virtual queues stable and at the same time performs arbitrarily close to the optimal performance.
- generalize our algorithm to the multihop scenario and develop distributed algorithms.

B. Virtual Queues

We define $\tilde{q}_d$ and $\tilde{q}_b$ as the virtual data and battery queues. The virtual queues evolve according to the following Lindley's queue evolution equations:

$$\tilde{q}_d(t+1) = \left(\left(\tilde{q}_d(t) - \eta_d R(t)\right)^+ - \mu\big(P(t)\big) + R(t) + I(t)\right)^+, \tag{11}$$

$$\tilde{q}_b(t+1) = \left(\left(\tilde{q}_b(t) - \eta_o\right)^+ + P(t) - r(t) + M(t) + I_o(t)\right)^+, \tag{12}$$

where $I(t) = (\mu(P(t)) - q_d(t))^+$ is the amount of transmitted idle packets when there is no enough data to transmit using the allocated energy, $M(t) = (q_b(t) - P(t) + r(t) - B_b)^+$ is the amount of missed replenishing energy due to full battery when $B_b < \infty$, and $I_o(t)$ is defined in Equation (10). Note that if $B_b < \infty$, $M(t) = 0$ and Equation (12) reduces to

$$\tilde{q}_b(t+1) = \left(\left(\tilde{q}_b(t) - \eta_o\right)^+ + P(t) - r(t) + I_o(t)\right)^+. \tag{13}$$

Without loss of generality, the initial state $\tilde{q}_b(0)$ and $\tilde{q}_d(0)$ can be set to be zero. The following proposition shows that if the virtual queues $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are all strongly stable, p_d and p_o are guaranteed to meet their constraints.

Proposition 1: If the virtual queues $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are both strongly stable, i.e.,

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \tilde{q}_d(t) < \infty,$$

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} \tilde{q}_b(t) < \infty,$$

$$\limsup_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T-1} q_b(t) < \infty,$$

then $p_d \leq \eta_d$ and $p_o \leq \eta_o$.

The proof of this result can be found in Appendix A. Next, we present our scheme.

III. Joint Rate Control and Power Allocation Algorithm

A. Algorithm

The algorithm consists of two components: a *rate control* component and a *power allocation* component. Both components are *index policies*, i.e., the solutions are memoryless

and they depend on the instantaneous values of the system variables.

Rate Control (RC):
We define $0 < V < \infty$ to be the control parameter of our algorithm. Let $Q_d(t) = q_d(t)$ when $B_d = \infty$, and let $Q_d(t) = \tilde{q}_d(t)$ when $B_d < \infty$. If $Q_d(t) \leq \frac{V}{2}$, the transmitting node chooses to sense all the available data, i.e., $R(t) = A(t)$. Otherwise, $R(t) = 0$.

Power Allocation (PA):
Solve the following optimization problem

$$\max_{P(t)\in\Pi(t)} Q_d(t)\mu\big(P(t)\big) - \tilde{q}_b(t)P(t), \tag{14}$$

where $\Pi(t) = \{P(t) : 0 \leq P(t) \leq \min[q_b(t), P_{\text{peak}}]\}$ is a compact and nonempty set. Allocate $P(t)$.

The set, $\Pi(t)$ of possible power allocations guarantees Constraint (d) on $P(t)$ in Problem (A). If $\mu(\cdot)$ is concave[1], the objective function is a concave function of $P(t)$. Consequently, *PA* solves a simple convex optimization problem in each time slot. The positive term $Q_d(t)\mu(P(t))$ can be viewed as the utility of allocating power $P(t)$ and the term $\tilde{q}_b(t)P(t)$ can be viewed as its associated cost. When the control parameter V is chosen to be large, $Q_d(t)$ tends to be large according to *RC*, and *PA* tries to allocate higher power $P(t)$ to increase the utility, whereas, when the virtual battery queue length $\tilde{q}_b(t)$ is large, *PA* avoids allocating a high amount of power to reduce the cost. Thus, this index policy of *PA* can be viewed as a *greedy profit maximization* scheme.

B. Performance Analysis

Recall that $A(t)$ is the available amount of sensing data and $R(t)$ is the actual amount of data the transmitting node chooses to sense. Clearly, using the rate controller, we make sure that the data queue remains within a certain bound and this has a positive effect on the battery as well, since a certain portion of the data packets are not allowed into the transmitting node. The natural question one would ask here is, whether our rate controller rejects too many packets in the first place to *synthetically* meet the constraints. In the following theorem, we show that this is not the case. Indeed, if there exists a solution, $\lambda^* = \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R^*(t)$ to Problem (A) for the exogenous process $\{A(t),\ t \geq 0\}$ and the replenishment process $\{r(t),\ t \geq 0\}$, then the sensing rate associated with *RC* and *PA* can be made asymptotically closer to λ^* by increasing the control parameter V with increasing B_d and B_b. We use the notation $y = O(x)$ to represent y going to 0 as x goes to 0.

Theorem 1: If the following conditions hold:
(1) $\mu(\cdot)$ is concave on $\Re^+ \bigcup \{0\}$, and its slope at 0 satisfies[1] $0 \leq \beta = \mu'(0) < \infty$,
(2) $r(t) > 0$, for all $t \geq 0$, and $\bar{r} = \limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} r(t) < P_{\text{peak}}$,
(3) A feasible solution to Problem (A) exists and the optimal instantaneous sensing rate is $R^*(t)$,
then the joint power allocation and admission control algorithm (with *RC* and *PA*) achieves:

$$Q_d(t) \leq \frac{V}{2} + A_{\max}, \quad \forall\ t \geq 0 \tag{15}$$

$$\tilde{q}_b(t) \leq \beta\left(\frac{V}{2} + A_{\max}\right), \quad \forall\ t \geq 0 \tag{16}$$

$$q_b(t) < \infty, \quad \forall\ t \geq 0 \tag{17}$$

$$\liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R(t) \geq \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} R^*(t) - O(\frac{1}{V}) - \eta_o(\mu_{\max} + \beta) - g(V, B_d, B_b), \tag{18}$$

where $\mu_{\max} = \mu(P_{\text{peak}})$ is the upper bound for the transmission rate, and

$$g(V, B_d, B_b) = \begin{cases} 0, & \text{if } B_d < \infty,\ B_b < \infty \\ O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}), & \text{if } B_d = \infty,\ B_b < \infty \\ O(\frac{(\frac{V}{2} - B_d)^+}{V}), & \text{if } B_d < \infty,\ B_b = \infty \\ O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}) + O(\frac{(\frac{V}{2} - B_d)^+}{V}), & \\ & \text{if } B_d < \infty,\ B_b < \infty. \end{cases}$$

The proof of Theorem 1 can be found in Appendix B. From Equation (15), Equation (16), and Equation (17), the virtual queues $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and the actual battery queue $q_b(t)$ are all strongly stable. Thus, by Proposition 1, $p_d \leq \eta_d$ and $p_o \leq \eta_o$. In Theorem 1, V is a finite tunable approximation parameter that controls the efficiency of the algorithm. Observe Equation (18), which compares the performance of our algorithm with that of the optimal solution of Problem (A), the term $\eta_o(\mu_{\max} + \beta)$ captures the influence of battery outage, and it is small since the battery outage threshold η_o is usually set to be very small to avoid network disconnection. Function $g(V, B_d, B_b)$ represents the asymptotical property of the gap. If both battery and the data buffers are of infinite size, then the performance gap is identical to 0. Otherwise the performance gap is dictated by the finite (or the smaller) one of the buffers. One can observe that, by appropriately choosing the V-parameter, we can make the performance gap decay inversely proportional to the buffer sizes.

IV. Multihop Network Model

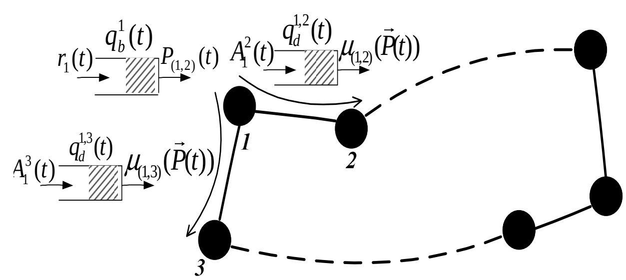


Fig. 2. Multihop Network Model

We consider a multihop wireless sensor network with N nodes and L links as illustrated in Figure 2. Each node

[1] For instance, consider the additive white Gaussian noise channel capacity, $\mu(P) = \log(1 + P/N_0)$, where N_0 is the two sided noise power spectral density.

$n \in \mathcal{N} = \{1, 2, \ldots, N\}$ is attached to power sources for replenishment. Let $A_n^e(t)$ and $R_n^e(t)$ denote the amount of available data for sensing and the actual amount of sensed data, to node n that are destined to node e in slot t. We assume that each node n maintains an infinite data buffer with state $q_d^{n,e}(t)$ for flows destined to e, and also maintains a finite battery buffer with size B_b^n and state $q_b^n(t)$ (We focus on infinite data buffer and finite battery buffer in order to emphasize the difference from single link to multihop. For other combinations of data and battery buffer sizes, then extension can be made similarly without too much effort). Let $r_n(t)$ denote the replenishment at node n in time slot t. The transmit power is chosen to be $P_l(t)$ over link l. In the formulation, we assume that the power the receiving node consumes to receive and decode the packet is identical to $P_l(t)$ as well. The sole reason for this is simplicity and the generalization to the asymmetric case is straightforward. We use the node-exclusive interference model. Under this model, a node can only receive from or transmit to at most one node at any time slot. In each time slot t, with the assigned power $P_l(t)$, the achieved data rate at link l is $\mu_l(P_l(t))$ in that time slot, where the rate function $\mu_l(\cdot)$ is a non-decreasing, concave and differentiable function satisfying $\mu_l(0) = 0$. Let p_o^n be the frequency of visits to the zero battery state for node n. Let Ω_n and Θ_n denote the set of directed links originated from node n and terminate at node n, respectively. We say $\vec{P} = [P_1(t) \cdots P_L(t)]$ satisfies the node-exclusive model if $P_l(t) > 0$ for some $l \in \Omega_n \cup \Theta_n$, then $P_{l'} = 0$ for all $l' \in (\Omega_n \cup \Theta_n) \setminus \{l\}$. In a multihop network, we formulate the joint queue and energy management problem as follows:

$$(B) \qquad \max_{\vec{P}, \vec{R}} \liminf_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \sum_{n, e \in \mathcal{N}} R_n^e(t)$$

s.t. $\vec{P}(t)$ satisfies the node-exclusive model, (19)

$$q_d^{n,e}(t+1) \le \Big(q_d^{n,e}(t) - \sum_{l \in \Omega_n} \mu_l^e\big(P_l(t)\big)\Big)^+ + R_n^e(t) + \sum_{l \in \Theta_n} \mu_l^e(P_l(t)), \quad n \ne e, \tag{20}$$

$$q_b^n(t+1) = \min\left[q_b^n(t) - \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) + r_n(t), B_b^n\right], \tag{21}$$

$$0 \le \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) \le q_b^n(t),\ 0 \le P_l(t) \le P_{\text{peak}}, \tag{22}$$

$$\sum_{e=1}^{N} \mu_l^e\big(P_l(t)\big) = \mu_l\big(P_l(t)\big),\ R_n^e(t) \le A_n^e(t), \tag{23}$$

$$\limsup_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} q_d^{n,e}(t) < \infty, \quad n \ne e, \tag{24}$$

$$p_o^n \le \eta_o^n, \tag{25}$$

where η_o^n is the desired upper bound for p_o^n, $\vec{P}(t)$ is the power assignment vector for all links in slot t, $\vec{P}$ is the power assignment for all links over all time slots, and $\vec{R}$ is the actual sensing data vector for all node-destination pairs over all time slots. In Problem (B), the objective is to maximize the long-term average total sensing rate for all nodes destined to all destinations.

In Problem (B), (19) is the interference constraint. Constraints (20) and (21) describe how the data and battery queues evolve, respectively. Note that the destination node of each flow does not need to maintain a data buffer for that flow, as indicated in (20). Constraints (22) are the energy conservation equations stating that we cannot oversubscribe the energy that is unavailable in the battery nor can we exceed the peak power level. Constraints (23) are the rate conservation equations that bound the actual amount of sensed data $R_n^e(t)$ by the available amount of data $A_n^e(t)$, and share the transmission rate of a link among all the destinations in slot t. Constraint (24) is the QoS constraint for data queue: we need to keep all the data queues stable. Constraint (25) is the battery QoS constraint of the desired battery discharge rate η_o^n.

Similarly, we define virtual queues for all $n \in \mathcal{N}$

$$\tilde{q}_b^n(t+1) = \bigg(\Big(\tilde{q}_b^n(t) - \eta_o^n\Big)^+ + \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) - r_n(t) + M_n(t) + I_o^n(t)\bigg)^+, \tag{26}$$

where $M_n(t) = \big(q_b^n(t) - \sum_{l \in \Omega_n \cup \Theta_n} P_l(t) + r_n(t) - B_b^n\big)^+$ is the amount of missed replenishment and

$$I_o^n(t) = \textit{indicator that battery state '0' is visited from higher states in slot } t \textit{ for node } n$$
$$= \begin{cases} 0 & \text{if } \sum_{l \in \Omega_n \bigcup \Theta_n} P_l(t) = 0 \text{ or } \sum_{l \in \Omega_n \bigcup \Theta_n} P_l(t) < q_b^n(t) \\ 1 & \text{otherwise} \end{cases}$$

Next, we generalize the main results and algorithms *RC* and *PA* to the multihop scenario.

Corollary 1: If all the virtual battery queues $\tilde{q}_b^n(t)$, $\forall n \in \mathcal{N}$ are strongly stable, we have $p_o^n \le \eta_o^n$, $\forall n \in \mathcal{N}$.
The proof is identical to the single hop scenario and can be found in Appendix C. We give the algorithm in the following section.

V. Joint Rate Control, Power Allocation and Routing Algorithm for Multihop Networks

The joint rate control, power allocation and routing algorithm for multihop networks can either be implemented in a centralized or distributed manner. For the centralized solution, we use the classical Maximal Weighted Matching (MWM) based algorithm and for the distributed algorithm, we can use the Maximal Matching (MM) based algorithm [17].

Multihop Rate Control (MRC):
Depending on whether Maximum Weight Matching or Maximal Matching is employed by the scheduler, there is a slight difference in the implementation of MRC:
Maximum Weighted Matching (MWM): If $q_d^{n,e}(t) \le \frac{V}{2}$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.

Maximal Matching (MM): If $q_d^{n,e}(t) \le V$, node n chooses to sense all the available data packets, i.e., $R_n^e(t) = A_n^e(t)$; otherwise, reject all the arrivals, i.e., $R_n^e(t) = 0$.

Multihop Power Allocation (MPA):
In order to ensure the stability of the actual and virtual queues, we try to make no node transfer data of a flow to a relay node that is not the destination of that flow unless the differential backlog of that flow is no less than a fixed value γ.

Let tran(l) and rec(l) denote the transmitting and receiving node of link l, respectively. We first define

$$\gamma_l^e = \begin{cases} \gamma & \text{if rec}(l) \neq e \\ 0 & \text{otherwise} \end{cases}$$

where $\gamma > 0$ is some constant. Let $e_l(t) = \arg\max_e \left\{ q_d^{\text{tran}(l),e}(t) - q_d^{\text{rec}(l),e}(t) - \gamma_l^e \right\}$ be the flow on link l that has the maximal modified differential backlog, and $w_l(t) = \max\left[q_d^{\text{tran}(l),e_l(t)}(t) - q_d^{\text{rec}(l),e_l(t)}(t) - \gamma_l^{e_l(t)}, 0 \right]$.

For each link l, solve

$$\max_{P_l(t)\in\Pi_l(t)} w_l(t)\mu_l\big(P_l(t)\big) - \Big(\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t)\Big)P_l(t) \tag{27}$$

where $\Pi_l(t) = \left\{ P_l(t) : 0 \le P_l(t) \le \min\left[q_b^{\text{tran}(l)}(t), q_b^{\text{rec}(l)}(t), P_{\text{peak}} \right] \right\}$. This ensures that no node can transfer a flow to a relay node that is not the destination of the flow unless the differential backlog of the data queues between these two nodes is no less than γ.

Let $P_l(t)$ be the solution. With the calculated power $P_l(t)$, let $W_l(t) = w_l(t)\mu_l\big(P_l(t)\big) - \Big(\tilde{q}_b^{\text{tran}(l)}(t) + \tilde{q}_b^{\text{rec}(l)}(t)\Big)P_l(t)$ be the weight on link l.

For the whole network, we either use the MWM or MM as described below. We will analyze the performance of our solution when using each algorithm.

Maximum Weighted Matching Algorithm: link l has weight $W_l(t)$, then the weight of a matching $\mathcal{M}$ is $W_{\mathcal{M}}(t) = \sum_{l\in\mathcal{M}} W_l(t)$. The network chooses a maximum weighted matching in a centralized manner, the links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Maximal Matching Algorithm: the network chooses a maximal matching in a fully distributed manner as in [17], the links in the chosen matching become active with the calculated transmitting power, and other links are not activated.

Multihop Routing:
When $w_l(t) > 0$, transmit for flow that is destined to $e_l(t)$ with rate $\mu_l\big(P_l(t)\big)$, i.e., $\mu_l^{e_l(t)}\big(P_l(t)\big) = \mu_l\big(P_l(t)\big)$ and $\mu_l^e\big(P_l(t)\big) = 0,\ \forall e \neq e_l(t)$.
Note that *MRC* and routing can be done by each node independently.

In order to avoid the trivial case that there is superfluous amount of replenishment to support the amount of available sensing data. We say the processes $\{\vec{A}(t),\ \forall t \ge 0\}$ and $\{\vec{r}(t),\ \forall t \ge 0\}$ are feasible if a feasible solution to Problem (B) exists and by supporting the optimal sensing rate, the resulted battery queue remains bounded even with an infinite battery buffer. We then give our main theorem for the multihop scenario:

Theorem 2: If
(1) $\mu_l(\cdot)$ is concave on $\Re^+ \bigcup\{0\}$, and its slope at 0 satisfies $0 \le \beta = \mu_l'(0) < \infty,\ \forall l \in \mathcal{L}$,
(2) $\forall n \in \mathcal{N}$: $r_n(t) > 0,\ \forall t \ge 0$,
(3) $\{\vec{A}(t),\ \forall t \ge 0\}$ and $\{\vec{r}(t),\ \forall t \ge 0\}$ are feasible, and the optimal instantaneous sensing rate vector is $\vec{R}^*(t)$,

then the maximum weighted matching based joint rate control *MRC*, power allocation *MPA*, and routing algorithm achieves:

$$q_d^{n,e}(t) \le \frac{V}{2} + A_{\max}, \tag{28}$$

$$\tilde{q}_b^n(t) \le \beta\Big(\frac{V}{2} + A_{\max}\Big), \tag{29}$$

$$\begin{aligned} &\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^e(t) \right\} \\ &\ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^{e*}(t) - \eta_o^n(\mu_{\max}+\beta) \right. \\ &\qquad \left. -O\big(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}\big) \right\} - O\big(\frac{1}{V}\big), \end{aligned} \tag{30}$$

and the maximal matching based joint rate control *MRC*, power allocation *MPA*, and routing algorithm achieves:

$$q_d^{n,e}(t) \le V + A_{\max}, \tag{31}$$

$$\tilde{q}_b^n(t) \le \beta(V + A_{\max}), \tag{32}$$

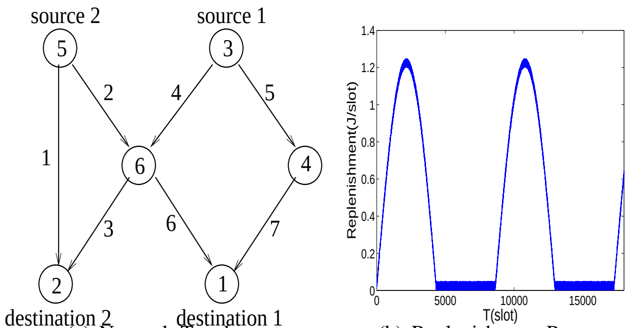
$$\begin{aligned} &\sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^e(t) \right\} \\ &\ge \sum_n \left\{ \liminf_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1}\sum_e \frac{R_n^{e*}(t)}{2} - \eta_o^n(\mu_{\max}+\beta) \right. \\ &\qquad \left. -O\big(\frac{(\beta V - B_b^n)^+}{V}\big) \right\} - O\big(\frac{1}{V}\big). \end{aligned} \tag{33}$$

The proof of Theorem 2 can be found in Appendix D. The results can be interpreted similarly as Theorem 1.

VI. Numerical Example

We consider a network topology, shown in Figure 3(a). There are 6 nodes, 7 links, and 2 flows with source-destination pair $(3,1)$ and $(5,2)$, respectively. There are $T = 10^6$ time slots, each of which is 10 secs long. We use the rate power function $\mu_l(P_l) = 10\log_2(1 + \frac{g_l P_l}{N_l})$ packets/slot $\forall l \in \mathcal{L}$. Let the power of the background noise $N_l = 1.6\times 10^{-14}W,\ \forall l \in \mathcal{L}$, and the channel gains $g_l = 1.6\times 10^{-13},\ \forall l \in \mathcal{L}$. Each node is equipped with an infinite data buffer for each flow through it, and a battery buffer of size $B_b = 800J$. The number of arrivals $A_n^e(t),\ t \ge 0$, for all nodes and flows, are modeled as independent Poisson random variables with mean 20 packets/slot and $A_{\max} = 31$ packets/slot. The replenishment process is a random process that is a periodic deterministic waveform plus independent Gaussian noise, as

shown in Figure 3(b) (The cycles imitate the daily solar cycles for a solar battery). We also set η_o^n, the threshold of battery outage probability to 0.03 for all $n \in \mathcal{N}$.



(a) Network Topology (b) Replenishment Process

Fig. 3. Settings

We choose different values of the control coefficient V for the proposed algorithm and compare the results with the optimal value[2]. From Figure 4(a), we see that as V increases, the average total sensing rate keeps increasing and gets closer to the optimal value, which is consistent with Equation (30). From Figure 4(b), we see that as V increases, the average data queue length (we here only plot the data queue length of node 2 for flow 1) keeps increasing but is upper bounded by the bound we get in Equation (28). From Figure 4(c) we observe that the battery discharge probability (we only plot for node 1) increases to the threshold as V increases. Thus, increasing V can increase the average total sensing rate, but the cost is increasing the average data queue length and battery discharge probability of at least one node.

Similarly, from Figure 5(a), we see that as V increases, the average total sensing rate keeps increasing and gets closer to some value that is larger than half of the optimal value, which is consistent with the result given by Equation (33). From Figure 5(b), we see that as V increases, the average data queue length (we here only plot the data queue length of node 3 for flow 1) keeps increasing but is upper bounded by the bound we get in Equation (31). From Figure 5(c) we observe that the battery discharge probability (we only plot for node 6) increases to the threshold as V increases.

VII. Conclusion

In this paper, we studied the problem of energy management in rechargeable wireless sensor networks. Our objective was to maximize the average data sensing rate subject to QoS constraints on both data and battery queues. We provided a simple and unified framework of joint rate control and power allocation for all combinations of finite and infinite data and battery buffer sizes. We showed through both analysis and simulation that the performance of our strategy is close to that of the optimal solution. We extended our algorithm to the multihop scenario and showed that simple extensions of our index schemes for single hop generalize to the multihop scenario with a similar, close-to-optimal performance. We developed a distributed joint rate control, power allocation and routing algorithm for multihop networks under node-exclusive interference model based on imperfect scheduling. Although we do not consider channel variations in this paper, taking channel variations into consideration is technically straightforward. Our multihop network formulation does not consider the fairness among links, however, the opportunistic scheduling framework [18] used to resolve the fairness issue can be applied in this context. For future work, we are extending this framework to the general interference model where the rate power function is not concave.

[2]The optimal value can be obtained by using dynamic programming. The details are omitted here.

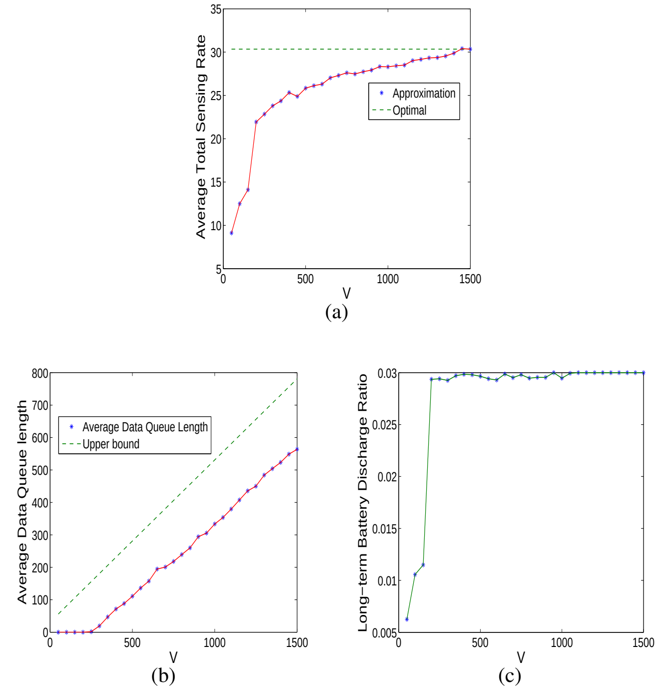


Fig. 4. Performance of the MWM based algorithm. Impact of the control parameter V on (a) the average total sensing rate, (b) average data queue length, and (c) the battery discharge probability

References

[1] A. Mainwaring, J. Polastre, R. Szewczyk, D. Culler, and J. Anderson, "Wireless Sensor Networks for Habitat Monitoring," in *Proc. of ACM WSNA*, September 2002, pp. 88–97.

[2] N. Xu, S. Rangwala, K. Chintalapudi, D. Ganesan, A. Broad, R. Govindan, , and D. Estrin, "A Wireless Sensor Network for Structural Monitoring," in *Proc. of SenSys*, 2004, pp. 13–24.

[3] K. Martinez, R. Ong, , and J. Hart, "Glacsweb: a Sensor Network for Hostile Environments," in *Proc. of the 1st IEEE Communications Society Conference on Sensor and Ad Hoc Communications and Networks*, Santa Clara, CA, 2004, pp. 81–87.

[4] J. Paradiso and M. Feldmeier, "A Compact, Wireless, Self-Powered Pushbutton Controller," in *Proc. of the 3rd International Conference on Ubiquitous Computing*, 2001, pp. 299–304.

[5] W. Weber, "Ambient Intelligence: Industrial Research on A Visionary Concept," in *Proc. of the 2003 International Symposium on Low Power Electronics and Design*, 2003, pp. 247–251.

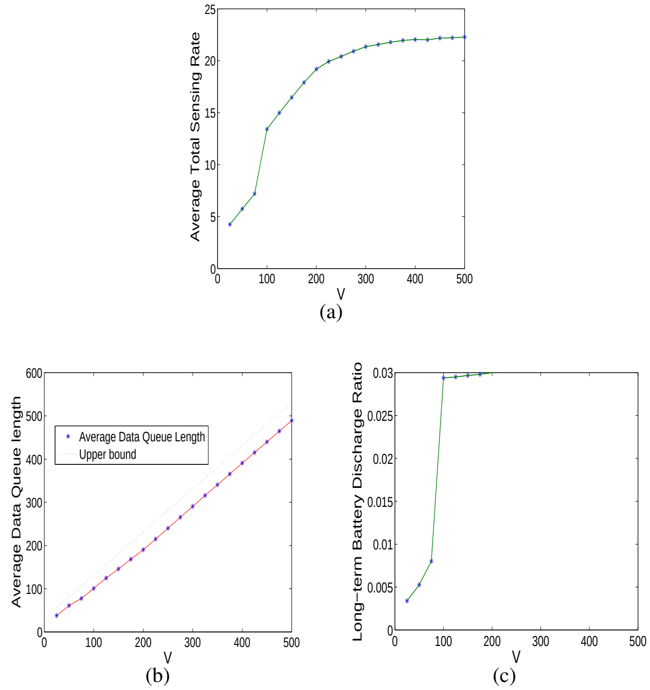


Fig. 5. Performance of the MM based algorithm. Impact of the control parameter V on (a) the average total sensing rate, (b) average data queue length, and (c) the battery discharge probability

[6] K. Kar, A. Krishnamurthy, and N. Jaggi, “Dynamic Node Activation in Networks of Rechargeable Sensors,” *IEEE/ACM Transactions on Networking*, vol. 14, pp. 15–26, 2006.

[7] K. Fan, Z. Zheng, and P. Sinha, “Steady and Fair Rate Allocation for Rechargeable Sensors in Perpetual Sensor Networks,” in *Proc. of the 6th ACM Conference on Embedded Network Sensor Systems*, 2008, pp. 239–252.

[8] L. Lin, N. Shroff, and R. Srikant, “Asymptotically Optimal Energy-Aware Routing for Multihop Wireless Networks with Renewable Energy Sources,” *IEEE/ACM Transactions on Networking*, vol. 15, no. 5, pp. 1021–1034, October 2007.

[9] A. Kansal, J. Hsu, S. Zahedi, and M. Srivastava, “Power Management in Energy Harvesting Sensor Networks,” *ACM Transactions on Embedded Computing Systems*, vol. 6, no. 4, September 2007.

[10] D. Niyato, E. Hossain, and A. Fallahi, “Sleep and Wakeup Strategies in Solar-Powered Wireless Sensor/Mesh Networks: Performance Analysis and Optimization,” *IEEE Transactions on Mobile Computing*, vol. 6, no. 2, pp. 221–236, February 2007.

[11] C. Vigorito, D. Ganesan, and A. Barto, “Adaptive Control of Duty Cycling in Energy-Harvesting Wireless Sensor Networks,” in *Proc. of IEEE SECON*, 2007.

[12] M. Gatzianas, L. Georgiadis, and L. Tassiulas, “Control of Wireless Networks with Rechargeable Batteries,” *IEEE Transactions on Wireless Communications*, vol. 9, no. 2, pp. 581–593, February 2010.

[13] Z. Mao, C. Koksal, and N. Shroff, “Resource Allocation in Sensor Networks with Renewable Energy,” in *Proc. of the 19th International Conference on Computer Communication Networks*, 2010.

[14] M. Neely, “Energy Optimal Control for Time Varying Wireless Networks,” *IEEE Transactions on Information Theory*, vol. 52, no. 7, pp. 2915–2934, July 2006.

[15] N. Salodkar, A. Bhorkar, A. Karandikar, and V. Borkar, “An On-Line Learning Algorithm for Energy Efficient Delay Constrained Scheduling over a Fading Channel,” *IEEE Journal on Selected Areas in Communications*, May 2008.

[16] N. Salodkar, A. Karandikar, and V. Borkar, “Power Efficient Scheduling under Delay Constraints over Multi-user Wireless Channels,” *IEEE Transactions on Mobile Computing*, 2007.

[17] X. Lin and N. Shroff, “The Impact of Imperfect Scheduling on Cross-Layer Congestion Control in Wireless Networks,” *IEEE/ACM Transactions on Networking*, vol. 14, no. 2, pp. 302–315, April 2006.

[18] X. Liu, E. Chong, and N. Shroff, “A Framework for Opportunistic Scheduling in Wireless Networks,” *Computer Networks*, vol. 41, no. 4, pp. 451–474, March 2003.

Appendix

A. Proof of Proposition 1

Using the idea similar to [14], we have the fact that if any queue represented with $Q(t)$ is strongly stable, then $\limsup_{T\to\infty}\frac{Q(T)}{T} = 0$. Hence, if $\tilde{q}_d(t)$, $\tilde{q}_b(t)$ and $q_b(t)$ are strongly stable, then $\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T} = 0$, $\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T} = 0$ and $\limsup_{T\to\infty}\frac{q_b(T)}{T} = 0$.

From Equation (11), we have $\tilde{q}_d(t+1) \geq \tilde{q}_d(t) - \eta_d R(t) + D(t) - D(t) - \mu(P(t)) + R(t) + I(t)$. Note that $q_d(t+1) = q_d(t) - \mu(P(t)) + I(t) + R(t) - D(t)$. By adding from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$ on both sides, we have

$$\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T} \geq \lim_{T\to\infty}\frac{\tilde{q}_d(0)}{T} - \eta_d \limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t) + \limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}D(t) + \lim_{T\to\infty}\frac{q_d(0)-q_d(T)}{T}.$$

Since $\limsup_{T\to\infty}\frac{\tilde{q}_d(T)}{T} = 0$, $\lim_{T\to\infty}\frac{\tilde{q}_d(0)}{T} = 0$, $\lim_{T\to\infty}\frac{q_d(0)}{T} = 0$ and $\lim_{T\to\infty}\frac{q_d(T)}{T} = 0$, so we get $p_o = \limsup_{T\to\infty}\frac{\sum_{t=0}^{T-1}D(t)}{\sum_{t=0}^{T-1}R(t)} \leq \eta_d$.

Similarly, from Equation (12), we have $\tilde{q}_b(t+1) \geq \tilde{q}_b(t) - \eta_o + P(t) - r(t) + M(t) + I_o(t)$. Note that $q_b(t+1) = q_b(t) - P(t) + r(t) - M(t)$. By adding from 0 to $T-1$, dividing by T and taking $\limsup_{T\to\infty}$ on both sides, we have

$$\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T} \geq \lim_{T\to\infty}\frac{\tilde{q}_b(0)}{T} - \eta_o + \limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_o(t) + \lim_{T\to\infty}\frac{q_b(0)-q_b(T)}{T}.$$

Since $\limsup_{T\to\infty}\frac{\tilde{q}_b(T)}{T} = 0$, $\lim_{T\to\infty}\frac{\tilde{q}_b(0)}{T} = 0$, $\lim_{T\to\infty}\frac{q_b(0)}{T} = 0$ and $\lim_{T\to\infty}\frac{q_b(T)}{T} = 0$, so we get $p_o = \limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_o(t) \leq \eta_o$. ■

B. Proof of Theorem 1

Proof of Equation (15): Note that $I(t) \leq \mu(P(t))$ and $R(t) \leq A_{\max}$. The rate allocation unit *RC* is chosen to satisfy Equation (15).

Proof of Equation (16): Since $\mu(\cdot)$ is concave on $\Re^+ \bigcup \{0\}$, we have $\mu(P(t)) \leq \mu(0) + \beta P(t)$ for $P(t) \in \Pi(t)$, $\forall t \geq 0$, where $0 \leq \beta = \mu'(0) < \infty$. Then, $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) \leq Q_d(t)\mu(0) + \beta Q_d(t)P(t) - \tilde{q}_b(t)P(t)$ where $P(t)$ is the solution of *PA*.

If $\beta Q_d(t)P(t) - \tilde{q}_b(t)P(t) < 0$, then we get $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) < Q_d(t)\mu(0)$. However, *PA* chooses $P(t)$ that maximizes $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)$ which means $Q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) \geq Q_d(t)\mu(0)$ since $0 \in \Pi(t)$. Then we must have $\beta Q_d(t)P(t) - \tilde{q}_b(t)P(t) \geq 0$, i.e., $\beta Q_d(t)P(t) \geq \tilde{q}_b(t)P(t)$. Thus, if $P(t) > 0$, $\tilde{q}_b(t) \leq \beta Q_d(t) \leq \beta(\frac{V}{2} + A_{\max})$; if

$P(t) = 0$, by Equation (10), $I_o(t) = 0$, and we also have $M(t) \leq r(t)$, then $\tilde{q}_b(t)$ does not increase anyway. Therefore, $\tilde{q}_b(t) \leq \beta(\frac{V}{2} + A_{\max})$ for all t, which is Equation (16).

Proof of Equation (17): Equation (17) is trivial when $B_b < \infty$. We only need to consider the scenario when $B_b = \infty$.

First, we provide a rough idea of the proof: by exploring the relations between $\tilde{q}_b(t)$ and $q_b(t)$, we notice that as $q_b(t)$ increases from 0 to at most $r_{\max} + \beta(\frac{V}{2} + A_{\max})$, $\tilde{q}_b(t)$ will hit zero at some slot. Once $\tilde{q}_b(t)$ becomes zero, *PA* either drains the battery queue or allocates P_{peak}. Since $P_{\text{peak}} > \bar{r}$, $q_b(t)$ will either be under $r_{\max}$ or start to decrease after finite number of slots. We now give the proof details.

Without loss of generality, let $q_b(0) = 0$. We have the following cases:
i) if $P(t) \geq r(t)$, $I_o(t) = 0$ and $\tilde{q}_b(t) > 0$, then $q_b(t+1) \leq q_b(t)$ and $\tilde{q}_b(t+1) - \tilde{q}_b(t) \leq q_b(t) - q_b(t+1)$, i.e., even if $\tilde{q}_b(t)$ increases, the increment is no larger than the decrement of $q_b(t)$;
ii) if $P(t) < r(t)$, $I_o(t) = 0$, then if $\tilde{q}_b(t+1) > 0$, $\tilde{q}_b(t) - \tilde{q}_b(t+1) \geq q_b(t+1) - q_b(t)$, i.e., the decrement of $\tilde{q}_b(t)$ is no less than the increment of $q_b(t)$, else if $\tilde{q}_b(t+1) = 0$, it goes to case iv);
iii) if $I_o(t) = 1$, then $q_b(t+1) = r(t) \leq r_{\max}$ by definition of discharging event Equation (10);
iv) if $\tilde{q}_b(t) = 0$, by Equation (14), *PA* chooses $P(t) = \min[q_b(t), P_{\text{peak}}]$, then either $q_b(t+1) = r(t) \leq r_{\max}$, or $q_b(t+1) = q_b(t) - P_{\text{peak}} + r(t)$. For the latter case, if $P_{\text{peak}} > r(t)$, then the battery queue state decrease and $\tilde{q}_b(t+1) = q_b(t) - q_b(t+1) = P_{\text{peak}} - r(t) > 0$; if $P_{\text{peak}} \leq r(t)$, then $\tilde{q}_b(t) = 0$ and case iv) continues. However, $P_{\text{peak}} > \bar{r}$, which means after at most $K < \infty$ slots, $r(t+K) < P_{\text{peak}}$ and the battery queue state starts to decrease.

Combining the above discussion with Equation (16), we have that $\forall t \geq 0$, there $\exists t_1(t), t_2(t) \geq 0$ such that

$$\begin{aligned} q_b(t) &\leq \min[q_b(0), r_{\max}] + |\tilde{q}_b(t_1(t)) - \tilde{q}_b(t_2(t))| \\ &\quad + K(r_{\max} - P_{\text{peak}})^+ \\ &\leq (K+1)r_{\max} + \beta(\frac{V}{2} + A_{\max}) < \infty, \end{aligned}$$

which is Equation (17).

Proof of Equation (18): We define the Lyapunov function $L(Q_d(t), \tilde{q}_b(t)) = Q_d^2(t) + \tilde{q}_b^2(t)$, and $\Delta(Q_d(t), \tilde{q}_b(t)) = L(Q_d(t+1), \tilde{q}_b(t+1)) - L(Q_d(t), \tilde{q}_b(t))$.
I) $B_d = \infty$ *and* $B_b = \infty$.
From Equation (13), we have $\tilde{q}_b^2(t+1) \leq (\tilde{q}_b(t) - \eta_o)^2 + (I_o(t) + P(t) - r(t))^2 + 2(\tilde{q}_b(t) - \eta_o)^+(I_o(t) + P(t) - r(t))$. Also from the data queue dynamics, we have $q_d^2(t+1) \leq q_d^2(t) + \mu^2(P(t)) + R^2(t) + 2q_d(t)R(t) - 2q_d(t)\mu(P(t))$, then

$$\begin{aligned} \Delta &= \Delta(q_d(t), \tilde{q}_b(t)) \\ &\leq \mu^2(P(t)) + R^2(t) + 2q_d(t)R(t) - 2q_d(t)\mu(P(t)) + (1+ \\ &\quad P_{\text{peak}})^2 + r_{\max}^2 + \eta_o^2 + 2\eta_o r_{\max} + 2\tilde{q}_b(t)(I_o(t) + P(t) - \\ &\quad r(t)) \\ &\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu_{\max}^2 + A_{\max}^2(t) + \\ &\quad VR(t) + 2\tilde{q}_b(t)(I_o(t) - r(t)) + 2\left[q_d(t) - V/2\right]R(t) - \\ &\quad 2\left[q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)\right]. \end{aligned}$$

It is apparent that *RC* is trying to minimize the term $[q_d(t) - V/2]\,R(t)$, and *PA* is trying to maximize the value of the term $[q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)]$. Since the optimal solution for Problem (A) may not be unique, we let $\mathcal{P}^*$ be the optimal solution set and $P^* \in \mathcal{P}^*$ be any optimal solution, for Problem (A). In time slot t, let $P_m(t)$ be the value that maximize the unconstrained objective function $q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)$. Since $\Pi(t) = \{P(t) : 0 \leq P(t) \leq \min[q_b(t), P_{\text{peak}}]\}$, only when $q_b(t) \leq P_{\text{peak}}$ and $P_m(t), P^*(t) \notin \Pi(t)$, we may have $q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t) \leq q_d(t)\mu(P^*(t)) - \tilde{q}_b(t)P^*(t)$. Thus, we have

$$\begin{aligned} \Delta &\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu_{\max}^2 + A_{\max}^2 + V(\\ &\quad R(t) - R^*(t)) + 2q_d(t)[R^*(t) - \mu(P^*(t))] + 2\tilde{q}_b(t)I_o(t) \\ &\quad + 2\tilde{q}_b(t)[P^*(t) - r(t)] + 2[q_d(t)(\mu(P^*(t)) - \mu(P(t))) + \\ &\quad \tilde{q}_b(t)(P(t) - P^*(t))]1_{[P_m(t), P^*(t) \notin \Pi(t)] \bigcap [q_b(t) \leq P_{\text{peak}}]}. \end{aligned}$$

When $P_m(t) \notin \Pi(t)$ and $q_b(t) \leq P_{\text{peak}}$, *PA* will allocate $q_b(t)$ amount of energy for transmission in order to maximize $q_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)$ within $\Pi(t)$. Under this situation, we must have $I_o(t) = 1$ since $r(t-1) > 0$. Further, $P^*(t) \notin \Pi(t)$ means $P^*(t) \geq P(t)$. Thus,

$$\begin{aligned} \Delta &\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu_{\max}^2 + A_{\max}^2 + V(\\ &\quad R(t) - R^*(t)) + 2q_d(t)[R^*(t) - \mu(P^*(t))] + 2\tilde{q}_b(t)[P^*(t) \\ &\quad - r(t)] + (V + 2A_{\max})(\beta + \mu_{\max})I_o(t). \end{aligned} \tag{34}$$

Lemma 1: Under the same conditions as those in Theorem 1, there exists an optimal policy under which the actual battery queue is strongly stable, i.e., $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_b^*(t) < \infty$.
Proof: Suppose there is an optimal policy under which $\limsup_{T\to\infty} \frac{1}{T}\sum_{t=0}^{T-1} q_b^*(t) = \infty$. This means there exists a subsequence of times $\{t_n\}$ such that $t_n \to \infty$ and $\frac{1}{t_n}\sum_{t=0}^{t_n-1} q_b^*(t) \to \infty$. In other words, $\forall Q > 0$, $\exists t_N \in \{t_n\}$ such that $\forall t_k \in \{t_n\}$ and $t_k \geq t_N$, we have $\frac{1}{t_k}\sum_{t=0}^{t_k-1} q_b^*(t) > Q$.

Let $N(i) = \min\{n : \frac{1}{n}\sum_{t=i}^{i+n-1} r(t) = \bar{r} + \epsilon(i)\}$, where $|\epsilon(i)| \geq 0$ can be chosen arbitrarily to ensure $0 \leq \bar{r} + \epsilon(i) \leq P_{\text{peak}}$ and $N(i) < \infty$, $\forall i \geq 0$. Let $Q^* = \max_{i\geq 0} N(i)[\bar{r} + \epsilon(i)] < \infty$.

Find Q and the associated t_N such that $\frac{1}{t_N}\sum_{t=0}^{t_N-1} q_b^*(t) > Q > Q^*$. Since $t_N < \infty$, $\exists 0 \leq T_0 \leq t_N - 1$ such that $q_b^*(T_0) > Q > Q^*$. Then, if we modify this optimal policy by keeping $P^*(t)$ unchanged $\forall t < T_0$, and letting $P^*(t) = \bar{r} + \epsilon(t) = \bar{r} + \epsilon(T_0)$, for $T_0 \leq t \leq T_0 + N(T_0)$, $P^*(t) =$

$\bar{r}+\epsilon(t)=\bar{r}+\epsilon(T_0+N(T_0))$, for $T_0+N(T_0)\le t\le T_0+N(T_0)+N(T_0+N(T_0))$ and so on. This ensures that the battery state will equal the starting state $q_b^*(T_0)$ after $N(T_0)$ slots and battery discharge does not happen in between. This process continues, as shown in Figure 6. Thus, we will not violate the constraint on the battery discharge ratio.

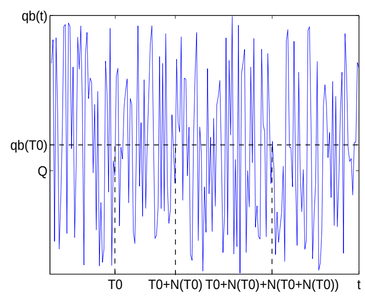


Fig. 6. Battery State Evolution under the Modified Optimal Algorithm

Under the modified policy, we have

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu(P^*(t))$$
$$=\liminf_{T\to\infty}\frac{1}{T}\left[\sum_{t=0}^{T_0-1}\mu(P^*(t))+\sum_{t=T_0}^{T-1}\mu(\bar{r}+\epsilon(t))\right]=\mu(\bar{r}+\epsilon) \tag{35}$$

By Jensen's inequality, we have

$$\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu(P^*(t))\le\mu(\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}P^*(t))$$
$$\le\mu(\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}r(t))=\mu(\bar{r}),$$

which means the feasible optimal sensing rate is strictly less than the maximum achievable service rate $\mu(\bar{r})$. From Equation (35), we can choose $\epsilon(t),\ \forall t\ge 0$ such that the modified policy provides a service rate which can support the original optimal sensing rate. In addition, we want to ensure that $0\le\bar{r}+\epsilon(i)\le P_{\text{peak}}$ and $N(i)<\infty,\ \forall i\ge 0$ as indicated before. A sequence $\epsilon(t),\ \forall t\ge 0$ that satisfies these conditions exists since $\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}r(t)=\bar{r}$, then the data queue is still strongly stable under the modified policy. Thus, the modified policy is still optimal.

Further, under the modified optimal policy, $q_b^*(t)\le r_{\max}T_0<\infty,\ \forall t<T_0$, and $q_b^*(t)<r_{\max}T_0+Nr_{\max}<\infty,\ \forall t\ge T_0$, where $N=\max_{i\ge T_0}N(i)<\infty$. Thus, the battery queue is strongly stable under the modified optimal policy, i.e., $\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}q_b^*(t)<\infty$. As a byproduct, $\limsup_{T\to\infty}\frac{q_b^*(T)}{T}=0$. ■

Note that $q_b^*(t+1)=q_b^*(t)-P^*(t)+r(t)$ for the optimal policy. By multiplying $\tilde{q}_b(t)$ on both sides and rearranging terms, we obtain $\tilde{q}_b(t)[P^*(t)-r(t)]=\tilde{q}_b(t)[q_b^*(t)-q_b^*(t+1)]$. Further, $-r_{\max}\le\tilde{q}_b(t)\le\beta(\frac{V}{2}+A_{\max})$. With Lemma 1, by summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$, we have

$$\begin{aligned}&\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_b(t)[P^*(t)-r(t)]\\&=\liminf_{T\to\infty}\frac{\tilde{q}_b(0)q_b^*(0)-\tilde{q}_b(T)q_b^*(T)}{T}\\&\quad+\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_b^*(t)(\tilde{q}_b(t)-\tilde{q}_b(t-1))\\&\le(1+P_{\text{peak}})\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_b^*(t),\end{aligned}\tag{36}$$

which is a finite constant that is not related to V.

Note that $q_d^*(t+1)=(q_d^*(t)-\mu(P^*(t)))^++R^*(t)\ge q_d^*(t)-\mu(P^*(t))+R^*(t)$. By multiplying both sides with $q_d(t)$ and rearranging terms, we obtain $q_d(t)(R^*(t)-\mu(P^*(t)))\le q_d(t)(q_d^*(t+1)-q_d^*(t))$. With the optimal policy, q_d^* is strongly stable. By summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$, we have

$$\begin{aligned}&\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}q_d(t)[R^*(t)-\mu(P^*(t))]\\&\le\liminf_{T\to\infty}\frac{q_d(T)q_d^*(T)-q_d(0)q_d^*(0)}{T}\\&\quad+\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^*(t)(q_d(t-1)-q_d(t))\\&\le A_{\max}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^*(t),\end{aligned}\tag{37}$$

which is a finite constant that is not related to V.

By summing from 0 to $T-1$, dividing by T and V, taking $\liminf_{T\to\infty}$ over Equation (34), combined with Equation (36), and Equation (37), we get

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t)\ge\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t)-\eta_o(\mu_{\max}+\beta)-O(\frac{1}{V}).$$

II) *$B_d=\infty$ and $B_b<\infty$.*
From Equation (12), we have $\tilde{q}_b^2(t+1)\le(\tilde{q}_b(t)-\eta_o)^2+(I_o(t)+P(t)-r(t)+M(t))^2+2(\tilde{q}_b(t)-\eta_o)^+(I_o(t)+P(t)-r(t)+M(t))$, then

$$\begin{aligned}\Delta&=\Delta(q_d(t),\tilde{q}_b(t))\\&\le(1+P_{\text{peak}})^2+\eta_o^2+r_{\max}^2+2\eta_or_{\max}+\mu^2(P(t))+R^2(t)\\&\quad+VR(t)+2\left[q_d(t)-V/2\right]R(t)-2[q_d(t)\mu(P(t))-\\&\quad\tilde{q}_b(t)P(t)]+2\tilde{q}_b(t)(I_o(t)-r(t)+M(t))\\&\le(1+P_{\text{peak}})^2+\eta_o^2+r_{\max}^2+2\eta_or_{\max}+\mu_{\max}^2+A_{\max}^2+\\&\quad V(R(t)-R^*(t))+2q_d(t)[R^*(t)-\mu(P^*(t))]+2\tilde{q}_b(t)[\\&\quad P^*(t)-r(t)+M^*(t)]+(V+2A_{\max})(\beta+\mu_{\max})I_o(t)\\&\quad+2\tilde{q}_b(t)M(t)\end{aligned}$$

Without loss of generality, let $q_b(0) = 0$. Similar to the proof of Equation (17), we have the following cases:
i) if $P(t) \geq r(t)$, $I_o(t) = 0$ and $\tilde{q}_b(t) > 0$, then $M(t) = 0$, $q_b(t+1) \leq q_b(t)$ and $\tilde{q}_b(t+1) - \tilde{q}_b(t) \leq q_b(t) - q_b(t+1)$, i.e., even if $\tilde{q}_b(t)$ increases, the increment is no larger than the decrement of $q_b(t)$;
ii) if $P(t) < r(t)$, $I_o(t) = 0$, then if $\tilde{q}_b(t+1) > 0$, $\tilde{q}_b(t) - \tilde{q}_b(t+1) = r(t) - P(t) - M(t) - \eta_o + (\eta_o - \tilde{q}_b(t))^+ \geq q_b(t+1) - q_b(t) = r(t) - P(t) - M(t)$, i.e., the decrement of $\tilde{q}_b(t)$ is no less than the increment of $q_b(t)$, else if $\tilde{q}_b(t+1) = 0$, it goes to case iv);
iii) if $I_o(t) = 1$, then $q_b(t+1) = \min[r(t), B_b]$ by definition of discharging event Equation (10). If $M(t) > 0$, then this means $r(t) > B_b$ and $-r(t) + M(t) = -B_b$;
iv) if $\tilde{q}_b(t) = 0$, then $\tilde{q}_b(t)M(t) = 0$ anyway.

Combine the above discussion with Equation (16), we have that $\forall t \geq 0$, if $M(t) > 0$ and $\tilde{q}_b(t) > 0$, we must have $\tilde{q}_b(t) \leq \beta(\frac{V}{2} + A_{\max}) + \max[r_{\max}, 1] - B_b$. Thus,

$$\begin{aligned}
&\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_b(t)M(t) \\
&\leq \frac{(\beta(\frac{V}{2} + A_{\max}) + \max[r_{\max}, 1] - B_b)^+ r_{\max}}{V} \\
&= O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}).
\end{aligned}$$

The remaining argument is similar to case I), and we obtain

$$\begin{aligned}
\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t) \geq \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t) - \eta_o(\mu_{\max} + \beta) \\
- O(\frac{1}{V}) - O(\frac{(\frac{\beta}{2}V - B_b)^+}{V}).
\end{aligned}$$

III) *$B_d < \infty$ and $B_b = \infty$.*
From Equation (13), we have $\tilde{q}_b^2(t+1) \leq (\tilde{q}_b(t) - \eta_o)^2 + (I_o(t)+P(t)-r(t))^2 + 2(\tilde{q}_b(t)-\eta_o)^+(I_o(t)+P(t)-r(t))$. Also from Equation (11), we have $\tilde{q}_d^2(t+1) \leq (\tilde{q}_d(t) - \eta_d R(t))^2 + (I(t) + R(t) - \mu(P(t)))^2 + 2(\tilde{q}_d(t) - \eta_d R(t))^+(I(t) + R(t) - \mu(P(t)))$, then

$$\begin{aligned}
\Delta &= \Delta(\tilde{q}_d(t), \tilde{q}_b(t)) \\
&\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu^2(P(t)) + (1 + \eta_d^2) \\
&\quad \cdot R^2(t) + VR(t) + 2\tilde{q}_b(t)(I_o(t) - r(t)) + 2\tilde{q}_d(t)I(t) + 2 \\
&\quad [(1 - \eta_d)\tilde{q}_d(t) - V/2]R(t) - 2[\tilde{q}_d(t)\mu(P(t)) - \tilde{q}_b(t)P(t)] \\
&\leq (1 + P_{\text{peak}})^2 + \eta_o^2 + r_{\max}^2 + 2\eta_o r_{\max} + \mu_{\max}^2 + (1 + \eta_d^2)A_{\max}^2 \\
&\quad + VR(t) - VR^*(t) + 2\tilde{q}_d(t)[R^*(t) - \mu(P^*(t)) + I^*(t) - \\
&\quad D^*(t)] + 2\tilde{q}_d(t)[D^*(t) - \eta_d R^*(t)] + 2\tilde{q}_b(t)[P^*(t) - r(t)] \\
&\quad + 2\tilde{q}_d(t)I(t) + (V + 2A_{\max})(\beta + \mu_{\max})I_o(t)
\end{aligned}$$

Without loss of generality, let $q_d(0) = B_d$. Similar to the proof of Equation (17), we have the following cases:
i) if $D(t) > 0$, then from $q_d(t+1) = (q_d(t) - \mu(P(t)))^+ + R(t) - D(t)$ and $I(t) = (\mu(P(t)) - q_d(t))^+$, $I(t)$ can be strictly positive only when $A(t) > B_d$. However, whenever $D(t) > 0$, $q_d(t+1) = B_d$.
ii) if $D(t) = 0$, from $q_d(t+1) = q_d(t) - \mu(P(t)) + I(t) + R(t) - D(t)$ and $\tilde{q}_d(t+1) = ((\tilde{q}_d(t) - \eta_d R(t))^+ + D(t) - \mu(P(t)) + I(t) + R(t) - D(t))^+$, $\tilde{q}_d(t)$ decreases no slower and increases no faster than $q_d(t)$ until $\tilde{q}_d(t)$ hits zero.

Note that only when $q_d(t) < \mu_{\max}$, $I(t)$ may be strictly positive. Then combine with the above discussion, we have

$$\begin{aligned}
&\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_d(t)I(t) \\
&\leq \frac{(\frac{V}{2} + A_{\max} + \mu_{\max} - B_d)^+ \mu_{\max}}{V} \\
&= O(\frac{(\frac{V}{2} - B_d)^+}{V}).
\end{aligned}$$

Construct an auxiliary queue $\bar{q}^*(t)$ with the following evolution:

$$\bar{q}^*(t+1) = \bar{q}^*(t) - (\eta_d R^*(t) + \epsilon) + D^*(t),$$

where $\epsilon > 0$ can be arbitrarily small. Since for an optimal policy,

$$\begin{aligned}
\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}D^*(t) \leq \eta_d \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t) \\
< \limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}(\eta_d R^*(t) + \epsilon),
\end{aligned}$$

so $\bar{q}^*(t)$ is strongly stable.

Note that $\tilde{q}_d(t)[D^*(t) - \eta_d R^*(t)] = \tilde{q}_d(t)[\bar{q}^*(t+1) - \bar{q}^*(t) + \epsilon]$. By summing from 0 to $T-1$, dividing by T and V, and taking $\liminf_{T\to\infty}$, we have

$$\begin{aligned}
&\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\tilde{q}_d(t)[D^*(t) - \eta_d R^*(t)] \\
&\leq \frac{1}{V}\liminf_{T\to\infty}\frac{\tilde{q}_d(T)\bar{q}^*(T) - \tilde{q}_d(0)\bar{q}^*(0)}{T} + \frac{\tilde{q}_d(t)}{V}\epsilon \\
&\quad + \frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\bar{q}^*(t)(\tilde{q}_d(t-1) - \tilde{q}_d(t)) \\
&\leq \frac{1}{V}A_{\max}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\bar{q}^*(t) + \frac{\epsilon}{V}(\frac{V}{2} + A_{\max}) \\
&= O(\frac{1}{V}) + \frac{\epsilon}{2}.
\end{aligned}$$

The remaining argument is similar to case I). Further, by letting $\epsilon \to 0$, we obtain

$$\begin{aligned}
\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t) \geq \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t) - \eta_o(\mu_{\max} + \beta) \\
- O(\frac{1}{V}) - O(\frac{(\frac{V}{2} - B_d)^+}{V}).
\end{aligned}$$

IV) *$B_d < \infty$ and $B_b < \infty$.*

Simply by combing case II) and case III), we obtain

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R(t)\geq\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}R^*(t)-\eta_o(\mu_{\text{max}}+\beta)\\-O(\frac{1}{V})-O(\frac{(\frac{\beta}{2}V-B_b)^+}{V})-\\O(\frac{(\frac{V}{2}-B_d)^+}{V}).\quad\blacksquare$$

C. Proof of Corollary 1

Similar to the proof of Proposition 1, we have the fact if $\tilde{q}_b^n(t)$ is strongly stable, then $\limsup_{T\to\infty}\frac{\tilde{q}_b^n(T)}{T}=0$. From Equation (26), we have $\tilde{q}_b^n(t+1)\geq\tilde{q}_b^n(t)-\eta_o^n+\sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t)-r_n(t)+M_n(t)+I_o^n(t)$. Note that $q_b^n(t+1)=q_b^n(t)-\sum_{l\in\Omega_n\bigcup\Theta_n}P_l(t)+r_n(t)-M_n(t)$. By summing from 0 to $T-1$, dividing by T and taking $\limsup$ of both sides, we have

$$\limsup_{T\to\infty}\frac{\tilde{q}_b^n(T)}{T}\geq\lim_{T\to\infty}\frac{\tilde{q}_b^n(0)}{T}-\eta_o^n+\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_o^n(t)\\+\lim_{T\to\infty}\frac{q_b^n(0)-q_b^n(T)}{T}.$$

Since $\limsup_{T\to\infty}\frac{\tilde{q}_b^n(T)}{T}=0$, $\limsup_{T\to\infty}\frac{\tilde{q}_b^n(0)}{T}=0$ $\limsup_{T\to\infty}\frac{q_b^n(T)}{T}=0$, and $\lim_{T\to\infty}\frac{\tilde{q}_b^n(0)}{T}=0$, so we get $p_o^n=\limsup_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_o^n(t)\leq\eta_o^n,\ \forall n\in\mathcal{N}$. $\blacksquare$

D. Proof of Theorem 2

Proof of Equation (28) and Equation (31): We prove Equation (28) by induction. Let $q_d^{\max}(t)$ be the maximum data queue length for all flows at slot t. Assume that $q_d^{\max}(t)\leq\frac{V}{2}+A_{\max}$ (holds for $t=0$ by letting $q_d^{n,e}(0)=0,\ \forall n,e\in\mathcal{N}$), need to show that it holds at slot $t+1$. Consider the data queue $q_d^{n,e}(t+1)$ maintained at any node n for flow destined to any node $e\neq n$ at slot $t+1$. If node n received data destined to e from other node m at slot t, then by the routing policy in Section V and definition of $w_{(m,n)}(t)$, $q_d^{m,e}(t)-q_d^{n,e}(t)>\gamma_{(m,n)}^e$, where (m,n) is the link from node m to node n. Choose γ such that the resulting backlog of the receiving node is not longer than that of the transmitting node (let $\mu_{\max}^{\text{in}}$ to be the maximum endogenous arrivals, then $\gamma=\mu_{\max}^{\text{in}}+A_{\max}$ satisfy this condition), we then have $q_d^{n,e}(t+1)\leq q_d^{n,e}(t)+\gamma$, then $q_d^{n,e}(t+1)\leq q_d^{m,e}(t)\leq q_d^{\max}(t)\leq\frac{V}{2}+A_{\max}$. If node n did not receive any data destined to e from other nodes, then it can only have exogenous arrivals. Clearly $q_d^{n,e}(t+1)\leq q_d^{n,e}(t)\leq\frac{V}{2}+A_{\max}$ if there were no exogenous arrivals. If there were exogenous arrivals, by *MRC* of Section V, we must have $q_d^{n,e}(t)\leq\frac{V}{2}$, then $q_d^{n,e}(t+1)\leq\frac{V}{2}+A_{\max}$. Thus, Equation (28) holds. Equation (31) can be shown using the same argument.

Proof of Equation (29) and Equation (32): Since $\mu_l(P_l(t))\leq\mu_l(0)+\beta P_l(t)$ for $P_l(t)\in\Pi_l(t),\ \forall t\geq0$, $w_l(t)\mu_l(P_l(t))-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})P_l(t)\leq w_l(t)\mu_l(0)+\beta w_l(t)P_l(t)-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})P_l(t)$ where $P_l(t)$ is the solution of Equation (27). If $\beta w_l(t)P_l(t)-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})P_l(t)<0$, then we get $w_l(t)\mu_l(P_l(t))-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})P_l(t)<w_l(t)\mu_l(0)$. However, solution of Equation (27) chooses $P_l(t)$ that maximizes $w_l(t)\mu_l(P_l(t))-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})P_l(t)$ which means $w_l(t)\mu_l(P_l(t))-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})P_l(t)\geq w_l(t)\mu_l(0)$ since $0\in\Pi_l(t)$. Then we must have $\beta w_l(t)P_l(t)-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})P_l(t)\geq0$. Using a similar argument as in the proof of Equation (16), we obtain $(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)})\leq\beta w_l(t)\leq\beta q_d^{\max}\leq\beta(\frac{V}{2}+A_{\max})$ for all t, and this holds for all $l\in\mathcal{L}$. If none of the incident links of a node is active, then by Equation (26), the virtual battery queue length does not increase anyway. Thus, Equation (29) is proved. Equation (32) can be shown using the same argument.

Proof of Equation (30): Define $L(\vec{q}_d(t),\vec{\tilde{q}}_b(t))=\sum_{n,e}(q_d^{n,e}(t))^2+\sum_n(\tilde{q}_b^n(t))^2$, then

$$\Delta(t)=L(\vec{q}_d(t+1),\vec{\tilde{q}}_b(t+1))-L(\vec{q}_d(t),\vec{\tilde{q}}_b(t))\\\leq\sum_{n,e}[A_{\max}^2+2A_{\max}\mu_{\max}+2\mu_{\max}^2]+\sum_n[(1+P_{\text{peak}})^2+\\(\eta_o^n)^2+r_{\max}^2+2\eta_o r_{\max}]+2\sum_{n,e}q_d^{n,e}(t)R_n^e(t)-2\sum_{l\in\mathcal{L}}\\\Big[(q_d^{\text{tran}(l),e_l(t)}(t)-q_d^{\text{rec}(l),e_l(t)}(t))\mu_l(P_l(t))-(\tilde{q}_b^{\text{tran}(l)}\\(t)+\tilde{q}_b^{\text{rec}(l)}(t))P_l(t)\Big]+2\sum_n\tilde{q}_b^n(t)\,[I_o^n(t)-r_n(t)+\\M_n(t)]$$

Note that if $q_d^{\text{tran}(l),e_l(t)}(t)-q_d^{\text{rec}(l),e_l(t)}(t)<\gamma_l^{e_l(t)}$, then $w_l(t)=0$ and $\mu_l(P_l(t))=0$, then $(q_d^{\text{tran}(l),e_l(t)}(t)-q_d^{\text{rec}(l),e_l(t)}(t))\mu_l(P_l(t))=w_l(t)\mu_l(P_l(t))$; otherwise, $q_d^{\text{tran}(l),e_l(t)}(t)-q_d^{\text{rec}(l),e_l(t)}(t)\geq w_l(t)$, then $(q_d^{\text{tran}(l),e_l(t)}(t)-q_d^{\text{rec}(l),e_l(t)}(t))\mu_l(P_l(t))\geq w_l(t)\mu_l(P_l(t))$. We then have

$$\Delta(t)\leq\sum_{n,e}[A_{\max}^2+2A_{\max}\mu_{\max}+2\mu_{\max}^2]+\sum_n[(1+P_{\text{peak}})^2+\\(\eta_o^n)^2+r_{\max}^2+2\eta_o r_{\max}]+2\sum_n\tilde{q}_b^n(t)[I_o^n(t)-r_n(t)+\\M_n(t)]+2\sum_{n,e}q_d^{n,e}(t)R_n^e(t)-2\sum_{l\in\mathcal{L}}[w_l(t)\mu_l(P_l(t))-\\(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)}(t))P_l(t)\Big]$$

Note that $\sum_{l\in\mathcal{L}}[w_l(t)\mu_l(P_l(t))-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)}(t))P_l(t)]=\sum_{l\in\mathcal{M}(t)}[w_l(t)\mu_l(P_l^s(t))-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)}(t))P_l^s(t)]\geq\sum_{l\in\mathcal{M}^*(t)}[w_l(t)\mu_l(P_l^s(t))-(\tilde{q}_b^{\text{tran}(l)}(t)+\tilde{q}_b^{\text{rec}(l)}(t))P_l^s(t)]$, where $\mathcal{M}(t)$ is the matching chosen by MWM algorithm and $\mathcal{M}^*(t)$ is the matching picked by optimal policy in slot t, and $P_l^s(t),\ \forall l\in\mathcal{L}$ are the suppositionally calculated power in *MPA*. Since the objective function of power allocation component is separable over links and for each link it is concave, using similar argument

as in single link case, we have

$$\Delta(t) \leq \sum_{n,e}[A_{\max}^2 + 2A_{\max}\mu_{\max} + 2\mu_{\max}^2] + \sum_n[(1+P_{\text{peak}})^2 + (\eta_o^n)^2 + r_{\max}^2 + 2\eta_o r_{\max} + (V+2A_{\max})(\beta+\mu_{\max})I_o^n(t)] + V\sum_{n,e}R_n^e(t) + 2\sum_{n,e}\left[q_d^{n,e}(t) - \frac{V}{2}\right]R_n^{e*}(t) - 2\sum_{n,e}\sum_{l\in\Omega_n}(q_d^{\text{tran}(l),e}(t) - q_d^{\text{rec}(l),e}(t) - \gamma)\mu_l^e(P_l^*(t)) + 2\sum_n \tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t) + M_n(t)\right]$$
$$\leq \sum_{n,e}[A_{\max}^2 + 2(A_{\max}+\gamma)\mu_{\max} + 2\mu_{\max}^2] + \sum_n[(1+P_{\text{peak}})^2 + (\eta_o^n)^2 + r_{\max}^2 + 2\eta_o r_{\max} + (V+2A_{\max})(\beta+\mu_{\max}) \cdot I_o^n(t)] + V\sum_{n,e}(R_n^e(t) - R_n^{e*}(t)) + 2\sum_n\tilde{q}_b^n(t)M_n(t) + 2\sum_{n,e}q_d^{n,e}(t)\left[R_n^{e*}(t) + \left(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\right)\mu_l^e(P_l^*(t))\right] + 2\sum_n\tilde{q}_b{}^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t)\right]. \tag{38}$$

Note that $q_b^{n*}(t+1) = q_b^{n*}(t) - \sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) + r_n(t) - M_n^*(t)$ for the optimal policy. By multiplying $\tilde{q}_b^n(t)$ for both sides and rearranging terms, we obtain $\tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t)\right] = \tilde{q}_b^n(t)[q_b^{n*}(t) - q_b^{n*}(t+1)]$. By summing from 0 to $T-1$, dividing by T and taking $\liminf_{T\to\infty}$, we have

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n\tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n}P_l^*(t) - r_n(t) + M_n^*(t)\right]$$
$$= \sum_n\liminf_{T\to\infty}\frac{\tilde{q}_b^n(0)q_b^{n*}(0) - \tilde{q}_b^n(T)q_b^{n*}(T)}{T} + \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_n q_b^{n*}(t)(\tilde{q}_b^n(t) - \tilde{q}_b^n(t-1))$$
$$\leq (1+P_{\text{peak}})\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\sum_n q_b^{n*}(t),$$

which is a finite constant that is not related to V and remains bounded as $B_b\to\infty$ since the replenishing process is feasible.

The argument of case II) in the proof of Theorem 1 can be directly applied here, and we have

$$\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n\tilde{q}_b^n(t)M_n(t) \leq \sum_n O(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V}).$$

Note that the data queue dynamics can be written as

$$q_d^{n,e}(t+1) = q_d^{n,e}(t) + R_n^e(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l(t)) - D_{\text{in}}^{n,e}(t) + D_{\text{out}}^{n,e}(t),$$

where $D_{\text{in}}^{n,e}(t)$ is the amount of overestimated endogenous arrivals to the queue $q_d^{n,e}$ since $\sum_{l\in\Theta_n}\mu_l^e(P_l(t))$ may be larger than the actual endogenous arrivals; $D_{\text{out}}^{n,e}(t)$ is the amount of overestimated departures since $\sum_{l\in\Omega_n}\mu_l^e(P_l(t))$ may be larger than the actual departures. Further, at slot t, if node n does not have endogenous arrivals for flow e, then $D_{\text{in}}^{n,e}(t) = 0$; if flow e is transferred from node n to node m, then $D_{\text{out}}^{n,e}(t) = D_{\text{in}}^{m,e}(t)$. By choosing $\gamma = A_{\max} + \mu_{\max}$, the resulting backlog of the receiving node is no longer than that of the transmitting node. We then have

$$\sum_{n,e}q_d^{n,e}(t)(q_d^{n,e*}(t+1) - q_d^{n,e*}(t))$$
$$= \sum_{n,e}q_d^{n,e}(t)\left[R_n^{e*}(t) - D_{\text{in}}^{n,e*}(t) + D_{\text{out}}^{n,e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))\right]$$
$$\geq \sum_{n,e}q_d^{n,e}(t)\left[R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))\right].$$

Since $q_d^{n,e}(t) - q_d^{n,e}(t+1) \leq A_{\max} + \mu_{\max}$, $\forall t\geq 0$, $\forall n,e\in\mathcal{N}$ and $q_d^{n,e*}$ are strongly stable $\forall n,e\in\mathcal{N}$, then

$$\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e}q_d^{n,e}(t)(q_d^{n,e*}(t+1) - q_d^{n,e*}(t))$$
$$\leq \sum_{n,e}\left[\lim_{T\to\infty}\frac{q_d^{n,e}(T)q_d^{n,e*}(T)}{T} + (A_{\max}+\mu_{\max}) \cdot \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^{n,e*}(t) - \lim_{T\to\infty}\frac{q_d^{n,e}(0)q_d^{n,e*}(0)}{T}\right]$$
$$\leq \sum_{n,e}\left[(\frac{V}{2} + A_{\max})\lim_{T\to\infty}\frac{q_d^{n,e*}(T)}{T} + (A_{\max}+\mu_{\max}) \cdot \liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^{n,e*}(t)\right]$$
$$\leq \sum_{n,e}(A_{\max}+\mu_{\max})\liminf_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}q_d^{n,e*}(t), \tag{39}$$

which is a finite constant that is not related to V. Thus, we have $\frac{1}{V}\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_{n,e}q_d^{n,e}(t)[R_n^{e*}(t) + \sum_{l\in\Theta_n}\mu_l^e(P_l^*(t)) - \sum_{l\in\Omega_n}\mu_l^e(P_l^*(t))] \leq O(\frac{1}{V})$. By summing from 0 to $T-1$, dividing by T and V, taking $\liminf_{T\to\infty}$ for Equation (38), we get

$$\sum_n\left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^e(t)\right\}$$
$$\geq \sum_n\left\{\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_e R_n^{e*}(t) - \eta_o^n(\mu_{\max}+\beta) + O(\frac{(\frac{\beta}{2}V - B_b^n)^+}{V})\right\} - O(\frac{1}{V}).$$

which is Equation (30).

Proof of Equation (33): Define $L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) = \frac{1}{2}\sum_{n,e}(q_d^{n,e}(t))^2 + \frac{1}{2}\sum_n(\tilde{q}_b^n(t))^2$, and then

$$\begin{aligned}\Delta(t) &= L(\vec{q}_d(t+1), \vec{\tilde{q}}_b(t+1)) - L(\vec{q}_d(t), \vec{\tilde{q}}_b(t)) \\ &\leq \sum_{n,e}[\frac{1}{2}A_{\text{max}}^2 + (A_{\text{max}}+\gamma)\mu_{\text{max}} + \mu_{\text{max}}^2] + \frac{1}{2}\sum_n[(1+ \\ &\quad P_{\text{peak}})^2 + (\eta_o^n)^2 + r_{\text{max}}^2 + 2\eta_o r_{\text{max}} + (V+2A_{\text{max}})(\beta+ \\ &\quad \mu_{\text{max}})I_o^n(t)] + V\sum_{n,e}R_n^e(t) + \frac{1}{2}\sum_{n,e}[q_d^{n,e}(t) - V]R_n^{e*}(t) \\ &\quad + \sum_n \tilde{q}_b^n(t)\left[\sum_{l\in\Omega_n\bigcup\Theta_n} P_l^*(t) - r_n(t) + M_n^*(t)\right] + \\ &\quad \sum_{n,e} q_d^{n,e}(t)\left[\left(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\right)\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]}\right] + \\ &\quad \sum_n \tilde{q}_b^n(t)M_n(t).\end{aligned}$$

Under the optimal policy, the amount of data destined to the same destination may be routed through different routes. We first logically decompose the flows that are differentiated by destination into flows that are differentiated by routes under this optimal policy. Let $\mathcal{F}$ denote the set of flows after the flow decomposition, then any flow $f \in \mathcal{F}$ has a fixed route. Let $e(f)$ denote the original flow from which flow f is decomposed, and $U_f(l)$ denote the one hop upstream link of link l in the route of flow f. Let $H^* = \{H_f^{l*}\}$ denote the routing matrix under the optimal policy, where $H_f^{l*} = 1$ means the data of flow f is routed through link l. We further logically decompose the data queues at nodes into data queues at links under this optimal policy. Since the sum queue length of the decomposed queues equals the original queue length, we have

$$\begin{aligned}&\sum_{n,e} q_d^{n,e}(t)\left[\frac{1}{2}R_n^{e*}(t) + \left(\sum_{l\in\Theta_n} - \sum_{l\in\Omega_n}\right)\mu_l^e(P_l^*(t))I_{[l\in\mathcal{M}(t)]}\right] \\ &= \sum_{l,f}\Big[\frac{1}{2}q_d^{\text{tran}(l),e(f)}(t)R_{\text{tran}(l)}^{f*}(t) - \Big(q_d^{\text{tran}(l),e(f)}(t) - \\ &\quad q_d^{\text{rec}(l),e(f)}(t)\Big)\mu_l^f(P_l^*(t))I_{[l\in\mathcal{M}(t)]}\Big] \\ &= \sum_{l,f} q_d^{\text{tran}(l),e(f)}(t)\Big[\frac{\sum_n R_n^{f*}(t)H_f^{l*}}{2} - \mu_l^f(P_l^*(t))I_{[l\in\mathcal{M}(t)]} \\ &\quad - \delta - \frac{\sum_n R_n^{f*}(t)H_f^{U_f(l)*}}{2} + \mu_{U_f(l)}^f(P_{U_f(l)}^*(t))I_{[U_f(l)\in\mathcal{M}(t)]} \\ &\quad + \delta\Big] \\ &\leq \sum_{l,f} q_d^{\text{tran}(l),e(f)}(t)\left(\bar{Q}_d^{l,f*}(t+1) - \bar{Q}_d^{l,f*}(t)\right) - \\ &\quad \sum_{l,f} q_d^{\text{tran}(l),e(f)}(t)\left(\bar{Q}_d^{U_f(l),f*}(t+1) - \bar{Q}_d^{U_f(l),f*}(t)\right),\end{aligned}$$

where $\bar{Q}_d^{l,f*}(t),\ \forall l \in \mathcal{L}, f \in \mathcal{F}$ has the following evolution:

$$\begin{aligned}\bar{Q}_d^{l,f*}(t+1) = &\left(\bar{Q}_d^{l,f*}(t) - \mu_l^f(P_l^*(t))I_{[l\in\mathcal{M}(t)]} - \delta\right)^+ \\ &+ \frac{1}{2}\sum_n R_n^{f*}(t)H_f^{l*}.\end{aligned} \tag{40}$$

Since $-\mu_{\text{max}} \leq q_d^{n,e}(t) - q_d^{n,e}(t+1) \leq A_{\text{max}} + \mu_{\text{max}},\ \forall t \geq 0,\ \forall n, e \in \mathcal{N}$, in order to apply the same idea of Equation (39) to bound the above term, we only need to show that all the accumulative data queues $\bar{Q}_d^{l,f*},\ \forall l \in \mathcal{L}, f \in \mathcal{F}$ are strongly stable. We know that all the data queues are strongly stable under the optimal policy queue evolution, then the queues are still strongly stable after flow and link decomposition. This further implies that the accumulative queues are strongly stable (note that the accumulative queues being stable does not imply each queue in the routes are stable), i.e., the queues with the following evolutions are strongly stable:

$$Q_d^{l,f*}(t+1) = \left(Q_d^{l,f*}(t) - \mu_l^f(P_l^*(t))\right)^+ + \sum_n R_n^{f*}(t)H_f^{l*}. \tag{41}$$

Let $\bar{\mu}_l^{f*} = \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^f(P_l^*(t))$ and $\lambda_l^{f*} = \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\sum_n R_n^{f*}(t)H_f^{l*}$. Without loss of generality, assume $\bar{\mu}_l^{f*} > 0$. We know the optimal sensing rate is feasible, so $\lambda_l^{f*} < \bar{\mu}_l^{f*}$. Then the queues with evolution Equation (41) being stable is equivalent to the normalized queues with the following evolutions being stable:

$$q_d^{l,f*}(t+1) = \left(q_d^{l,f*}(t) - 1\right)^+ + \frac{\lambda_l^{f*}}{\bar{\mu}_l^{f*}}. \tag{42}$$

Note that $I_{[l\in\mathcal{M}(t)]}$ is a random variable which is independent of $P_l^*(t)$. Thus, $I_{[l\in\mathcal{M}(t)]}$ and $\mu_l^f(P_l^*(t))$ are independent and then uncorrelated, so

$$\begin{aligned}&\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^f(P_l^*(t))I_{[l\in\mathcal{M}(t)]} \\ &= \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\mu_l^f(P_l^*(t))\liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}I_{[l\in\mathcal{M}(t)]} \\ &= \liminf_{T\to\infty}\frac{1}{T}\sum_{t=0}^{T-1}\bar{\mu}_l^{f*}I_{[l\in\mathcal{M}(t)]}.\end{aligned}$$

In order to show that all the queues with evolution Equation (40) are strongly stable, we only need to show all the normalized data queues under the MM based algorithm with the following queue evolutions are strongly stable:

$$\bar{q}_d^{l,f*}(t+1) = \left(\bar{q}_d^{l,f*}(t) - I_{[l\in\mathcal{M}(t)]}\right)^+ + \frac{\lambda_l^{f*}}{2\bar{\mu}_l^{f*}}. \tag{43}$$

Equation(42) implies Equation (43) by a similar argument and proof techniques as in [17]. ■